Condensation of Polyakov's loops and Chromomagnetic fields in QGP

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Abstract

In QCD, the deconfinement phase transition is accompanied by the creation of the $A_0 =$ const condensate and strong temperature dependent chromomagnetic H^3, H^8 and usual magnetic H^{em} fields.

A gauge invariance of the A_0 condensation is proven within the Nielsen identity method. It is shown that the effective action$W(A_0, \xi)$ in the background R_ξ gauge accounting for the one-loop, two-loop and plasmon diagram contributions satisfies the Nielsen identity. The A_0 and Polyakov's loop $\langle L \rangle$ are mutually related. We express $W(A_0, \xi)$ in terms of $\langle L \rangle$ and obtain the effective potential of order parameter $W(A_0^{cl})$ which is independent of ξ and has a nontrivial minimum position. Hence the A_0^{cl} condensate value is detected.

At this background, the color charges Q^3_{ind} and Q^8_{ind} are generated. They are temperature dependent and produce related color electric fields E^3_{color} and E^8_{color}.

The mechanism for stabilization of magnetic field at high temperature due to the A_0 presence is clarified. A number of applications of the condensation in high temperature QGP is discussed.

All these may serve as signals of the phase transitions - creation of quark-gluon plasma.

Outline

- New signals of Deconfinement PT
- QGP, A_0 condensation
- QGP, spontaneous magnetization
- Violation of Furry's theorem in QGP
- Induced charges $Q^3_{ind.}$, Q^8_{ind} and potentials $\bar{\phi}^3$, $\bar{\phi}^8$
- Photon dispersion equation
- Effective $\gamma\gamma g$ and g^3 vertexes
- Inelastic scattering of photons in QGP
- Conclusion

1 Deconfinement phase transition (DPT)

Investigations of deconfinement phase of QCD is a hot topic nowadays. Due to asymptotic freedom of non-Abelian gauge field interactions at high temperature $T \geq 150$ MeV quarks are deliberated from hadrons and new matter state - quark-gluon plasma (QGP) - is formed. At lower temperatures quarks are confined inside hadrons. The order parameter of the DPT is the Polaykov loop (PL)

$$P(\vec{x}) = T \exp\big[ig \int dx_4 A_0(\vec{x}, x_4)\big]. \tag{1}$$

It equal 0 at low temperature and $P \neq 0$ at $T > T_d$.

If $A_0(x_4) = const$

$A_0 \neq 0$ **is also the order parameter of the** DPT. The condensation of the A_0 was demonstrated in either lattice simulations or in analytic calculations. $A_0 \neq 0$ violates the $Z(3)$ and gauge symmetries.

Review paper O.A. Borisenko, J. Bohacik, V.V. Skalozub, A_0 **condensate in QCD, Fortschr. Phys. v. 43 (1995) 301.**

2 EP and Nielsen's identity

SU(2) gluodynamics in the background field $\bar{A}^a_\mu = A_0 \delta_{\mu 0} \delta^{a3} = const$ is described by the Lagrangian

$$L = \frac{1}{4}(G^a_{\mu\nu})^2 + \frac{1}{2\xi}[(\bar{D}_\mu A_\mu)^a]^2 - \bar{C}\bar{D}_\mu D_\mu C. \qquad (2)$$

The gauge field potential $A^a_\mu = Q^a_\mu + \bar{A}^a_\mu$ is decomposed in quantum and classical parts. The covariant derivative in Eq.(2) is $(\bar{D}_\mu A_\mu)^a = (\partial_\mu \delta^{ab} - g\epsilon^{abc}\bar{A}^c_\mu)A^b_\mu$, $G^a_{\mu\nu} = (\bar{D}_\mu Q_\nu)^a - (\bar{D}_\nu Q_\mu)^a - g\epsilon^{abc}Q^b_\mu Q^c_\nu, g$ is a coupling constant, internal index a = 1,2,3. The Lagrangian of ghost fields $\bar{C}, C$ is determined by the background covariant derivative $\bar{D}_\mu(\bar{A})$ and the total one $D_\mu(\bar{A} + Q)$.

We obtain the two-loop effective potential

$$
\begin{aligned}
W(x) &= W^{(1)}(x) + W^{(2)}(x), \qquad (3)\\
\beta^4 W^{(1)}(x) &= \frac{2}{3}\pi^2[B_4(0) + 2B_4(\frac{x}{2})],\\
\beta^4 W^{(2)}(x) &= \frac{1}{2}g^2[B_2^2(\frac{x}{2}) + 2B_2(0))B_2(\frac{x}{2})] + \frac{2}{3}g^2(1-\xi)B_3(\frac{x}{2})B_1(\frac{x}{2}),
\end{aligned}
$$

where $B_i(x)$ are Bernoulli's polynomials defined *modulo* 1 , $x = \frac{gA_0\beta}{\pi}$. In what follows we consider the interval $0 \leq x \leq 2$.

Let us investigate the minima of it:

$$
\begin{aligned}
\beta^4 W_{min} &= \beta^4 W(0) - \frac{1}{192\pi^2}(3-\xi)^2 g^4,\\
&x = g^2\frac{(3-\xi)}{8\pi^2}, \qquad (4)
\end{aligned}
$$

Hence the gauge invariance of the A_0 condensation phenomenon is questionable.

This problem was solved within Nielsen's identity method

Skalozub (1992), (2021).

Nielsen's identity for a general type effective potential reads

$$\delta' W(\phi) = W_{,i} \delta \chi^i(\bar{\phi}), \tag{5}$$

which describes a variation of $W(\phi)$ due to variation of the gauge fixing term $F^\alpha(\phi)$.

In field theory $\delta\chi^i$ is calculated from equation

$$\delta\chi^i = -\Big\langle D^i_\alpha(\phi)\Delta^\alpha_\beta(\phi)\delta' F^\beta(\phi)\Big\rangle, \tag{6}$$

Nielsen's identity for two-loop EP (3) is

$$\frac{dW}{d\xi} = \frac{\partial W^{(2)}}{\partial \xi} + \frac{\partial W^{(1)}}{\partial x}\frac{\partial x}{\partial \xi} = 0, \tag{7}$$

Eq.(7) states that $W(x,\xi)$ does not change along the characteristic curves

$$x = x' + \frac{g^2}{4\pi^2} B_1(\frac{x'}{2})(\xi - \zeta) \tag{8}$$

in the plain of variables (x,ξ), ζ is an arbitrary integration constant.

Along them a variation in ξ is compensated by the special variation of x'.

It was shown,

Skalozub. A_0 condensation, Nielsen's identity and effective potential of order parameter, Phys. Part. Nucl. Lett., v. 18 (2021) 738, that effectively to obtain ξ-independent EP expressed in the terms of Polaykov's loop it is sufficient to replace in (8) $x' \to x_{cl.}, \xi \to -1$. In particular, in (4).

We call it effective potential of order parameter $W_L(x_{cl})$.

Other important order parameter is the temperature dependent chromo (magnetic) fields $H(T) \neq 0$ spontaneously created in the volume of the QGP. This point will not be discussed in this talk. In the literature, numerous applications of the PL in the QGP have been discussed. The combinations of both $A_0 \neq 0$ and $H(T) \neq 0$ were also investigated.

Physically, condensation of gluon fields is due to asymptotic freedom. From very general principles of QFT it follows that the stability at high temperature, field strength, momentum, chemical potential is reflected in instability at small values of these variables. And wise-versa, zero-charge behavior at large values of variables is reflected in stability of a vacuum.

Condensation generates stability factors.

It was observed in the literature that A_0 is dominant at temperatures not much grater T_d. So, in what follows we consider this case.

We describe some new phenomena and effects taking place due to the A_0 presence.

Spontaneous vacuum magnetization at LHC

Recently (Skalozub, Minaiev (2018)) it was obtained that at LHC experiment energies the QGP should be spontaneously magnetized.

The strengths of the large scale temperature dependent chromomagnetic, $B_3(T)$, $B_8(T)$, and usual magnetic, $H(T)$, fields spontaneously generated after the DPT, were estimated.

The critical temperature for the magnetized plasma is found to be $T_d(H) \sim 110 - 120$ MeV. This is essentially lower compared to the zero field value $T_d(H = 0) \sim 160 - 180$ MeV usually discussed in the literature. Due to contribution of quarks, the color magnetic fields act as the sources generating H. The strengths of the fields are $B_3(T), B_8(T) \sim 10^{18} - 10^{19}G$, $H(T) \sim 10^{16} - 10^{17}G$ for temperatures $T \sim 160 - 220$ MeV.

The presence of strong large scale (color) magnetic fields modifies the spectrum of the (color) charged particles that influence various processes of interest.

3 QGP at A_0 condensate

Quarks interact with electromagnetic field and gluons according the form

$$L^{int.} = \bar{\psi}^a[\gamma_\mu(\partial_\mu\delta^{ab} - ie_f A_\mu\delta^{ab} - ig(Q_\mu\frac{\lambda}{2})^{ab}) - m_f\delta^{ab}]\psi^b, \qquad (9)$$

where A_μ is potential of electromagnetic fields, Q_μ is potential of gluon field, e_f is electric charge of quark with flavor f, m_f is quark mass, g is charge of strong interactions, a, b are color indexes.

Since quarks carry both electric and strong charges in the QGP the effective interactions of color and white objects are possible due to the quark virtual loops.

The A_0 is an element of the center $Z(3)$ of the $SU(3)$ group. When it is non zero,

both of these symmetries are broken.

The A_0 is a specific classical external field. It can be introduced by splitting

$Q^a_\mu = (A_0)^a_\mu + (Q^a_\mu)_{rad.}$. of the gluon field potential. In what follows we consider the case $(A_0)^a_\mu = (A_0)_\mu\delta^{a3}$. This is for short.

4 Violation of Furry's theorem in QGP

In the vacuum, Furry's theorem holds:

The amplitudes having odd number of photon(gluon) lines, generated by the fermion loops, equal zero.

It is the consequence of C-parity invariance. The contribution of particles cancels the contribution of antiparticles.

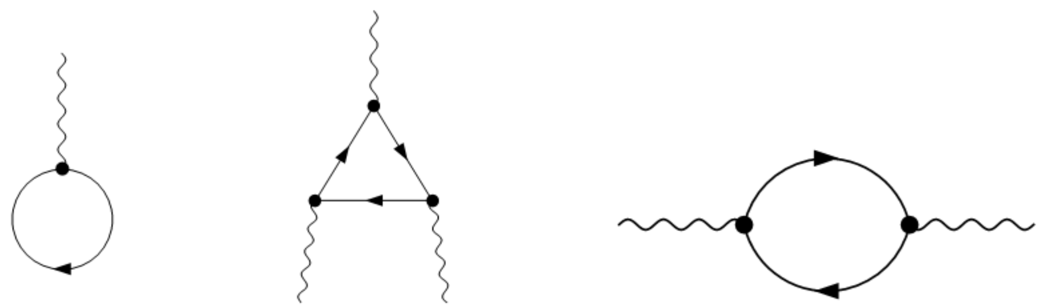
The presence of the A_0 condensate violates this symmetry. So that new type processes are permissible.

In particular,

the diagram with one gluon external line results in an induced color charge in the plasma. This may result in the scattering of quarks on this external charge.

Other interesting object is

Three line vertex - photon-photon-gluon - relates colored and white states. This is new type effective vertex which generates new observable processes - inelastic scattering of photons, splitting (dissociation) of gluons in two photons in the QGP. One of our goals is to calculate



this vertex and investigate these processes in the plasma.

These can be signals of the creation of QGP.

5 Gluon and photon spectra in QGP

Before doing that we have to detect the normal photon and gluon modes presented in the QGP with A_0. This can be done by solving the dispersion equations for these fields.

M. Bordag, V. Skalozub (2019)

Basically, in the plasma the spectra of the excitations can be obtained from the dispersion relations of the type

$$\omega^2 - \vec{k}^2 = Re\Pi(\omega, \vec{k}), \tag{10}$$

where ω and $\vec{k}$ are the frequency and the momentum of the modes.

In the QGP the transverse and the longitudinal excitations present. They are derived from relevant polarization tensors $\Pi(\omega, \vec{k})_T$ and $\Pi(\omega, \vec{k})_L$.

The expression for the photon polarization tensor reads

$$\Pi_{\mu\nu}(k) = -e^2 \sum_{p_4} \int \frac{d^3p}{(2\pi)^3\beta} Tr[\gamma_\mu \frac{(p+k)_\sigma\gamma_\sigma + m}{(p+k)^2+m^2} \gamma_\nu \frac{p_\rho\gamma_\rho + m}{p^2+m^2}]. \quad (11)$$

Here, imaginary time formalism is used. $\gamma_\mu, ...$ are the Dirac matrixes, $p_4 = 2\pi T(l+\frac{1}{2}) + A_0$, $k_\mu = (k_4 = 2\pi T(n), \vec{k})$, and $l, n = 0, \pm 1, \pm 2, ...$

Such type objects must be calculated in the gluon sector of the model.

As an example, we show the high temperature dispersion equation for the transversal plasma oscillations generated by the gluons

O. K. Kalashnikov, Progr. Theor. Phys. v. 92 (1994) 1207.:

$$(ik_4)^2 = g^2T^2\big[B_2(\frac{x}{2}) + B_2(0)\big]\xi^2\big(\frac{\xi^2}{\xi^2-1} - \frac{\xi}{2}\log\frac{\xi+1}{\xi-1}\big) + i\Gamma. \quad (12)$$

In this formula, $B_2(z) = z^2 - |z| + 1/6$ is the Bernoulli polynomial, $x = A_0/\pi T, \xi = (ik_4 + A_0)/|\vec{k}|$ and Γ is an imaginary part of the expression. It describes the damping of the plasma oscillations.

The similar expression have been obtained for longitudinal oscillations (plasmons) in the high temperature limit $T \to \infty$.

To find Dispersion relations we have to replace $ik_4 \to \omega$. In such a way all the quasi particle states of photons and gluons have been derived.

The A_0 condensate stabilizes the infrared behavior of the plasma and has a lover energy as compared to the empty vacuum case.

6 Induced charge in QGP

Generation of the strong charge due to one-line non-zero diagram.

I. Baranov, V. Skalozub (2018)

Its quark loop contribution can be calculate from the expression

$$Q^{quark}_{induced} = -g \sum_{p_4} \int \frac{d^3p}{(2\pi)^3\beta} Tr\gamma_4 [\frac{\lambda^3}{2} \frac{(p+k)_\sigma\gamma_\sigma + m_f}{(p+k)^2 + m_f^2}]. \qquad (13)$$

Here, the momentum $p = (p_4 = p_4 \pm A_0, \vec{p})$, $p_4 = 2\pi T(l + 1/2), l = 0, \pm 1, ..., \beta = 1/T$.

Similar expressions can be calculated from tadpole gluon diagram having charged gluon loop.

These also hold for the color charge Q_8.

The resulting induced charge changes the coupling constant of gluons in the QGP.

We obtain in the high temperature limit ($\beta \to 0$)

$$Q^{quark}_{3ind.} = gA_0\Big(\frac{T^2}{3} - \frac{m^3}{T} + O(1/T^3)\Big). \qquad (14)$$

In the presence of the induced charge the Slavnov-Taylor identity reads

$$\hat{p}_\mu \Pi^{\perp}_{\mu\nu}(\hat{p}_4, \vec{p}) = gJ^3_\nu. \qquad (15)$$

The induced current is

$$J^3_\nu = 2igQ_{3ind.}u_\nu, \qquad (16)$$

u_ν is plasma velocity.

7 Potentials of classical color fields

V. Skalozub (2019)

The induced color charges in the plasma result in the generation of classical gluon potentials. We introduce a simple model motivated by heavy-ion collisions.

We consider the QGP confined in the plate of the size L in z-axis direction and infinite in x-, y- directions. For this geometry, we calculate the classical potentials $\bar{\phi}^3 = G_4^3, \bar{\phi}^8 = G_4^8$ by solving the classical field equations for the gluon fields G_4^3, G_4^8 generated by the induced charges $Q^3_{ind.}, Q^8_{ind.}$. In doing so we take into consideration the results by

Kalashnikov (1994, 96) who calculated the gluon modes at the A_0 background. Either transversal or longitudinal modes were derived. For our problem, we are interested in the latter ones. The longitudinal modes of fields G_4^3, G_4^8 have temperature masses $\sim g^2T^2$. They are not affected by the background fields.

The classical potential $\bar{\phi}^3$ is calculated from the equation

$$[\frac{\partial^2}{\partial x_\mu^2} - m_D^2]\bar{\phi}^3 = -Q^3_{ind.}. \tag{17}$$

Similar equation is for $\bar{\phi}^8$.

Making Fourier's transformation to momentum k-space we derive the spectrum of modes, - $k_4^2 = k_x^2 + k_y^2 + k_z^2 + m_D^2$, where $k_z^2 = (\frac{2\pi}{L})^2 l^2$ and $l = 0, \pm 1, \pm 2, ...$ The discreteness of k_z is due to the periodic boundary condition for the plane: $\bar{\phi}^3(z) = \bar{\phi}^3(z + L)$. The general solution to Eq.(17) is

$$\bar{\phi}^3(x_4, \vec{x}) = d + a\ e^{-i(k_4 x_4 - \vec{k}\cdot\vec{x})} + b\ e^{i(k_4 x_4 - \vec{k}\cdot\vec{x})}. \tag{18}$$

At zero induced charge, d = 0, and we have two well known plasmon modes. In case of $Q^3_{ind.} \neq 0$, the values a, b, d calculated from the confinement boundary condition

$$\bar{\phi}^3(z = -\frac{L}{2}) = \bar{\phi}^3(z = \frac{L}{2}) = 0 \tag{19}$$

result in the expression

$$\bar{\phi}^3(z) = \frac{Q^3_{ind.}}{m_D^2}\left[1 - \frac{\cos(k_z z)}{\cos(k_z L/2)}\right]. \tag{20}$$

There are no dynamical plasmon states at all. This is the main observation. In the presence of the induced charges, the static classical color potentials (and, hence, fields) have to realize in the plasma.

By dimension analysis we have $\frac{Q^3_{ind.}}{m_D^2} \sim \frac{g A_0 T^2}{g^2 T^2}$ and $g A_0 \sim g^2 T$.

Hence, $\bar{\phi}^3(z) \sim cT$**,**

where $c \geq 0$ is a positive number!

For applications it is also necessary to get the Fourier's transform $\bar{\phi}^3(k)$ of the potential (20) to momentum space k. Fulfilling that for the interval of z $[-\frac{L}{2}, \frac{L}{2}]$ we obtain

$$\bar{\phi}^3(k) = \frac{Q^3_{ind.}L}{m_D^2} \; \frac{\sin(kL/2)}{(kL/2)} \frac{k_z^2}{k_z^2 - k^2}, \tag{21}$$

where the values of k_z are given after Eq.(17).

The energy for a one mode with momentum k_z is positive and equals to

$$E_l = \frac{(Q^3_{ind.})^2}{m_D^4} \frac{k_z^2}{2} L = \frac{(Q^3_{ind.})^2}{m_D^4} \frac{2\pi^2}{L} l^2. \tag{22}$$

The total energy is given by the sum over l of energies (22).

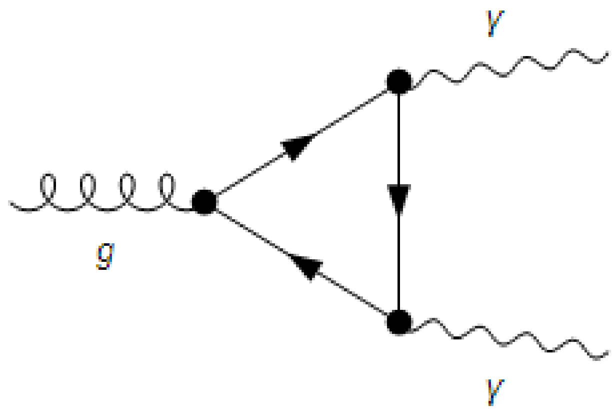
In the presence of the induced charges the static gluon potentials with positive energy should be generated. Dynamical longitudinal modes do not exist. This is the consequence of the condition Eq.(19).

Obvious that such a situation is independent of the form of the bag where the plasma is confined. In general, we have to expect that the color static potentials $\bar{\phi}^3$, $\bar{\phi}^8$ have to exist in the QGP and produce specific processes.

8 Effective $\gamma\gamma G$ vertexes in QGP

Explicit form for the photon-photon-gluon vertex, its dominant terms are

M. Bordag, V. Skalozub (2019)



$$\Pi_{\mu\nu\lambda}(k^1, k^2, k^3) = \delta(k^1 + k^2 + k^3)(-e^2 g\Lambda) \sum_{p_4} \int \frac{d^3p}{(2\pi)^3\beta} (\Gamma^{(1)}_{\mu\nu\lambda} + \Gamma^{(2)}_{\mu\nu\lambda}), \tag{23}$$

$\Lambda = -16 A_0 m_f^2$,

$$\Gamma^{(1)}_{\mu\nu\lambda} = \frac{\delta_{\mu\nu}\delta_{\lambda 4} + \delta_{\mu\lambda}\delta_{\nu 4} + \delta_{\lambda\nu}\delta_{\mu 4}}{d^2(p) d^2(p, k^1) d^2(p, k^3)}, \tag{24}$$

and

$$\Gamma^{(2)}_{\mu\nu\lambda} = \frac{-2S_{\mu\nu\lambda}}{d^2(p)d^2(p,k^1)d^2(p,k^3)}\Big(\frac{(p+k^3)_4}{d^2(p,k^3)} + \frac{(p-k^1)_4}{d^2(p,k^1)} + \frac{p_4}{d^2(p)}\Big), \quad (25)$$

where $d^2(p) = p^2 + m_f^2, d^2(p,k^1) = (p-k^1)^2 + m_f^2, d^2(p,k^3) = (p+k^3)^2 + m_f^2$,

$$S_{\mu\nu\lambda} = \delta_{\mu\nu}(p+k^1+k^3)_\lambda + \delta_{\lambda\nu}(p-k^1-k^3)_\mu + \delta_{\mu\lambda}(p-k^1+k^3)_\nu. \quad (26)$$

In the above formulas, k^1, k^3 are momenta of ingoing photons and $k^2 = -(k^1+k^2)$ is momentum of ingoing color neutral gluon $Q^{a=3}$.

All the other three-vertexes composing photons and gluons are zero.

So, we have a possibility for direct interaction of color and white world.

The most important points:

1. The vertex is not transversal

2. It relates transversal and longitudinal modes of photons and gluons

In particular, new phenomena such as scattering of photons on the QGP as an effective vertex become possible. All the necessary ingredients to investigate these are calculated. These are the spectra of photons and gluons in the QGP, the effective charges.

There are two sorts of the processes of interest:

1) Scattering of photons on the plasma as on the external filed generated due to quark current and induced color charge. Radiation of photon pairs from plasma.

2) Scattering on the real gluon excitations in the plasma.

In these processes the plasma exhibits itself via the effective vertex and therefore the inelastic (or even elastic) scattering may be realized. Specific values for these cases depend on the characteristics of QGP.

Scattering of photons in the QGP has to be estimated by two parameters - induced charge and deviation of of the photon beams from an initial direction.

Other important expected process is splitting of the gluon field G^3, G^8 generated by the induced charge $Q^3_{ind.}, Q^8_{ind.}$ in two photons which have to move along the direction of the plasma motion.

These processes are basically different from the scattering of photons on chaotically moving particles of usual plasma.

9 Conclusion

According to basic principles of QCD, the QGP has to be either magnetized with strong long range temperature dependent magnetic fields $B^3(T), B^8(T), H(T)$ (that lowers the deconfiniment transition temperature T_d) or charged with color induced charges $Q^3_{ind.}, Q^8_{ind.}$.

Due to violation of Furry's theorem, in the QGP new type phenomena have to be generated. Among them the deviation of the photon beam from its initial direction and the change of the frequency. Generation of induced color charges, gluon splitting in two photons. These are the distinguishable signals of the QGP creation.

In paper

M. Bordag, V. Skalozub. The effective potential of gluodynamics in the background of Polyakov loop and colormagnetic field. Eur. Phys. J C 82 (2022) 390 the analytic expression for the EP $W_L(H, A_0^{cl})$ which generalizes the expression (3) has been introduced. In case $H = 0$ it coincides with the latter one. I case $A_0^{cl} = 0$ it reproduces the known expression for the two-loop ffective potential of chromomagnetic fields. This potential opens prospects for description of new effecs and phenomena in QGP. These are problems for future.

THANK YOU FOR ATTENTION!