Динамічна генерація маси у тривимірній квантовій електродинаміці

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КВАНТОВА ТЕОРІЯ ПОЛЯ ТА КОСМОЛОГІЯ

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Outline

- Dynamical generation of electron mass in QED4 (P.I. Fomin)
- Gap generation in BCS and NJL models
- Quantum electrodynamics in $2 + 1$ dimensions
- QED_3 as a toy model for confinement
- Dynamical mass generation in QED_3
- Singularities of fermion self-energy in conventional perturbation theory

Dynamical generation of electron mass in QED4

Масса электрона, предистория.

Лоренц, Абрахам: классическая релятивистская модель электрона конечного радиуса r_0, где масса электрона равна энергии создаваемого им электрического поля

$$m = \frac{1}{4\pi}\frac{e^2}{r_0} \to \infty, \;\; r_0 \to 0 \quad (\frac{1}{r_0} = \Lambda).$$

В квантовой теории:

$$m = m_0 + \frac{3\alpha}{4\pi} m_0 \ln \frac{\Lambda^2}{m_0^2}$$ $-$теория возмущений (V. Weiskopff, 1934)

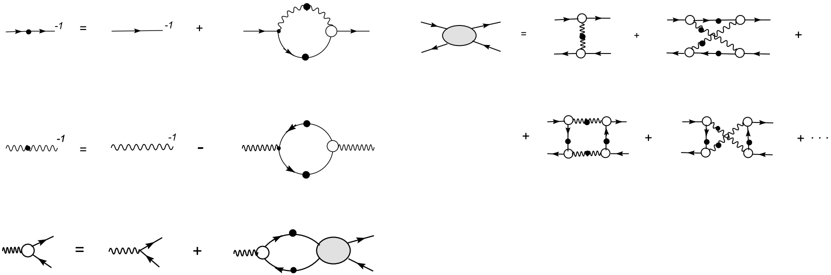
Самосогласованное уравнение для массы

$$m = m_0 + \frac{3\alpha}{4\pi} m \ln \frac{\Lambda^2}{m^2}$$

дает

$$m = \Lambda \exp\left(-3\pi/2\alpha\right), \quad m_0 = 0.$$

Уравнения Швингера-Дайсона



Ландау, Абрикосов, Халатников - 1954-1955

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}\left(i\gamma^\mu D_\mu - m_0\right)\psi, \; D_\mu = \partial_\mu + ie_0 A_\mu.$$

Расходимости содержатся в константах перенормировок электронного, Z_2, и фотонного, Z_3, пропагаторов, вершинной функции, Z_1, и затравочной массы m_0 $(Z_1 = Z_2)$.

С учетом 3-х вершин в уравнении для вершины Г ЛАХ получили в главном логарифмическом приближении (отсуммированы все члены ряда теории возмущений $(\alpha_0 \ln \frac{\Lambda^2}{m^2})^n$):

$$m_0 = m \left(1 - \frac{\alpha}{3\pi} \ln \frac{\Lambda^2}{m^2}\right)^{9/4}, \Lambda - \text{cuttof},$$

$$\alpha_0 = Z_3^{-1}\alpha = \frac{\alpha}{1 - \frac{\alpha}{3\pi} \ln \frac{\Lambda^2}{m^2}}, \quad \alpha = \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{m^2}},$$

при условии $\alpha_0 << 1, \frac{\alpha_0}{\pi} \ln \frac{\Lambda^2}{m^2} \lesssim 1.$
При снятии обрезания, $\Lambda \to \infty$, или $\alpha_0 \to \infty$ (полюс Ландау), или $\alpha \to 0$ (московский нуль).
Б.Л. Йоффе - Eur. Phys. J. H38, 83–135 (2013) (arxiv:1208.1386).
Затравочная масса уменьшается с ростом Λ! $m_0 = 0$?

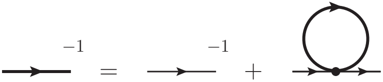
Lagrangian of the NJL model (Phys. Rev. 122, 345 (1961))

$$\mathcal{L} = \bar{\Psi} i\gamma^\mu \partial_\mu \Psi + \frac{G}{2}\left[(\bar{\Psi}\Psi)^2 + (\bar{\Psi} i\gamma^5 \Psi)^2\right]$$

is invariant under ordinary and chiral gauge transformations

$$\Psi \to e^{i\alpha}\Psi, \quad \Psi \to e^{i\alpha\gamma^5}\Psi, \quad \Longrightarrow$$
$$\partial_\mu(\bar{\psi}\gamma^\mu\psi) = 0, \quad \partial_\mu(\bar{\psi}\gamma^\mu\gamma^5\psi) = 0.$$

Chiral symmetry forbids mass generation in perturbation theory. The self-consistent Hartree-Fock equation for the fermion propagator $G(p) = [\gamma^\mu p_\mu - m]^{-1}$,



leads to the equation for dynamical mass (Λ is a cutoff):

$$m = G \int^{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{4m}{p^2+m^2} \Longrightarrow \frac{4\pi^2}{G\Lambda^2} = 1 - \frac{m^2}{\Lambda^2} \ln\left(1+\frac{\Lambda^2}{m^2}\right).$$

It has a nontrivial solution for m if the coupling $G > G_c = 4\pi^2/\Lambda^2$:

$$m^2 \simeq \Lambda^2 \frac{G-G_c}{G \ln \frac{G}{G-G_c}}, \quad G \gtrapprox G_c.$$

The vacuum hosts a non-vanishing chiral condensate $\langle \bar{\Psi}\Psi \rangle \neq 0$.
Chiral symmetry is spontaneously broken.
DOS $\nu(E) \sim E^2$ for massless particles vanishes at $E = 0$ - the reason for the existence of critical coupling G_c.

$$\Delta = \omega_D \exp\left(-2/g\nu_F\right), \ \nu(E_F) \sim \sqrt{E_F}, \text{ BCS superconductivity.}$$

ω_D - Debye frequency, g - electron-phonon coupling constant, ν_F - DOS on the Fermi surface. $E_F = 0$, the solution $\Delta \neq 0$ exists at $g > g_c = 2\pi^2/m^{3/2}\omega_D^{1/2}$!.

П.И. Фомин: $\alpha(\alpha L)^n$- приближение

$$\frac{m_0}{m} = \left(1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2}\right)^{9/4}\left[1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2} - \frac{3\alpha}{8\pi} + \frac{97}{66}\frac{\alpha^2}{\pi^2}\ln\frac{\Lambda^2}{m^2} + \frac{27}{16}\frac{\alpha}{\pi}\ln\left(1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2}\right)\right],$$

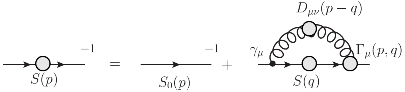
$$\frac{\alpha}{\alpha_0} = 1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2} + \frac{\alpha}{18\pi} + \frac{3\alpha}{4\pi}\ln\left(1 - \frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2}\right).$$

Для $m_0 = 0$ существует нетривиальное решение для динамической массы электрона неаналитическое по α:

$$m = \Lambda \exp\left(-\frac{3\pi}{2\alpha} + \frac{9}{8}\ln\frac{1}{\alpha} + c_1 + c_2\alpha + \dots\right), \ \alpha << 1.$$

При этом затравочная константа α_0 является конечной!

П.И. Фомин с сотрудниками (В.И. Трутень), работы 1967–1976 гг. (Харьков)



SDE for $S(p) = A(p^2)\hat{p} - B(p^2)^{-1}$.

For the dynamical mass function $B(p^2)$ in the ladder approximation, one gets the nonlinear equation (70th-80th:Maskawa, Nakajima, Fukuda, Kugo, Kiev group, Atkinson, Bardeen)

$$B(p^2) = \frac{3\alpha_0}{4\pi}\int_0^{\Lambda^2} \frac{dk^2 B(k^2)}{k^2 + B^2(k^2)}\left(\frac{\theta(p^2-k^2)}{p^2} + \frac{\theta(k^2-p^2)}{k^2}\right)$$

$$B(0) \equiv m, \quad m \simeq \Lambda \exp\left(-\frac{const}{\sqrt{\alpha_0 - \alpha_c}}\right), \; \alpha_c = \pi/3 \sim 1.$$

The solution combines features of a superconducting gap (non-analytical dependence on a coupling constant) and of the NJL gap (critical coupling). Taking into account the vacuum polarization:

$$m \simeq \Lambda\,(\alpha_0 - \alpha_c)^\beta,\; \beta \lesssim 1/2.$$

Experimental search for the strong coupling phase of QED:

GSI-Darmstadt in 80th (R.Peccei, Nature, 332, 492 (1988)).

Quantum electrodynamics in $2+1$ dimensions

Massless QED$_3$ in Euclidean space $\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}$:

$$S = \int d^3x \left[\bar{\psi}_i \gamma_\mu D_\mu \psi_i + \frac{1}{4} F_{\mu\nu}^2\right], \; D_\mu = \partial_\mu - ieA_\mu, \; i = 1, 2, ...N.$$

QED$_3$ is superrenormalizable with a mass scale $e^2N/8$ $([e] = (mass)^{1/2})$.

Large flavor symmetry $U(2N)$ with $(2N)^2$ generators

$$\frac{\lambda^a}{2}, \quad \frac{\lambda^a}{2i}\gamma^3, \quad \frac{\lambda^a}{2}\gamma^5, \quad \frac{\lambda^a}{2}\frac{[\gamma^3,\gamma^5]}{2}, \qquad a = 0, 1, \ldots N^2 - 1.$$

Adding a mass (gap) term $m\bar{\psi}\psi$ would reduce the $U(2N)$ symmetry down to $U(N)\times U(N)$ symmetry.

Applications in condensed matter physics:

- high-T$_c$ superconductivity
- planar antiferromagnets
- graphene.

QED_3 as a toy model for confinement

In the leading order in the $1/N$ expansion, the effective dimensionless coupling

$$\bar{\alpha}(p) = \frac{e^2}{p[1+\Pi(p)]}, \quad \Pi(p) = \frac{e^2 N}{8p}, \quad p = \sqrt{p^2},$$

$$\overline{\alpha}(p) = \frac{e^2}{p\,(1+\Pi(p^2))} = \begin{cases} \frac{e^2}{p} & p \gg e^2 N/8 \\ \frac{8}{N} & p \ll e^2 N/8 \end{cases} .$$

This gives rise to the renormalization-group β-function

$$\beta(\bar{\alpha}) \equiv p\frac{d\bar{\alpha}(p)}{dp} = -\bar{\alpha}\left(1 - \frac{N}{8}\bar{\alpha}\right),$$

which has the ultraviolet stable fixed point $\bar{\alpha} = 0$ at $p \to \infty$ (asymptotic freedom) and the infrared stable fixed point $\bar{\alpha} = 8/N$ at $p = 0$. The parameter $e^2 N/8$ plays a role similar to the QCD scale Λ_{QCD}, and the effective coupling $\bar{\alpha}(p)$ approaches zero at large momenta p.

Fixed points in non-abelian theories

Renormalization group equation for the running coupling constant in $SU(N_c)$ gauge theories with n_f quarks:

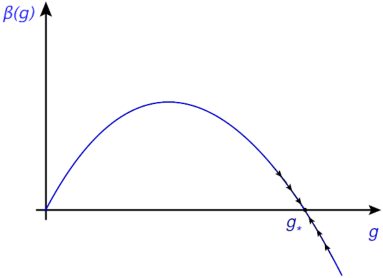
$$\mu\frac{d\alpha}{d\mu}=\beta(\alpha)=-\alpha^2[\underbrace{b}_{\text{1-loop}}+\underbrace{c\alpha}_{\text{2-loop}}+\underbrace{c_2\alpha^2}_{\text{3-loop}}+c_3\alpha^3+\ldots],\quad \alpha(\mu)=\frac{g_s^2}{4\pi}.$$

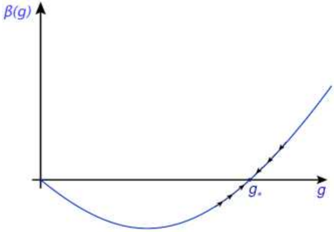
$$b=\frac{11N_c-2n_f}{6\pi},\quad c=\frac{1}{24\pi^2}\left(34N_c^2-10N_cn_f-3\frac{N_c^2-1}{N_c}n_f\right),$$

The beta-function is known up to six loops. The theory is asymptotically free if $b>0$ ($n_f<11N_c/2$). At two loops, the theory has an infrared stable, nontrivial fixed point

$$\alpha_*=-\frac{b}{c}>0,\quad \frac{11N_c}{2}>n_f>\frac{34N_c^3}{13N_c^2-3},\ (N_c=3:\ 16\geq n_f\geq 9)-$$

Belavin-Migdal-Caswell-Banks–Zaks fixed point.





Fixed point g_* of the beta-function $\beta(g)$ separates different phases of a theory.

Static interaction potential

$$V(r) = \int \frac{d^2p}{(2\pi)^2} e^{i\vec{p}\vec{r}} D_{00}(p_0 = 0, \vec{p}) = \int \frac{d^2p}{(2\pi)^2} \frac{e^{i\vec{p}\vec{r}}}{\vec{p}^2[1+\Pi(\vec{p}^2)]}$$

$$\sim \begin{cases} \frac{1}{r} & m = 0 \\ e^2 \ln r & m \neq 0 \end{cases} \quad \text{at large distances.}$$

If in the massless theory we decrease the fermion number N then the fixed point coupling α_* increases, and at some critical value of N_c the fermion mass is dynamically generated. Hence there will be a phase transition from a massless phase to the chirally asymmetric (massive) phase which is a confined phase.

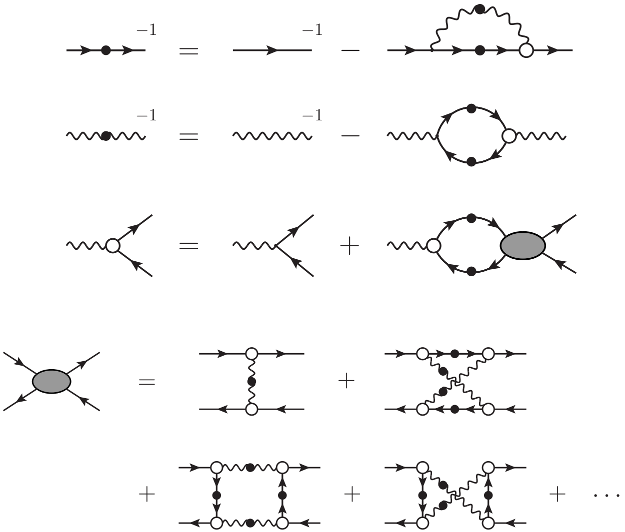
An important problem in QED_3 is to determine N_c and to characterize the nature of the phase transition.

Previous studies of mass generation in QED_3

R. Pisarski, Chiral symmetry breaking in three-dimensional electrodynamics, Phys. Rev. D 29, 2423 (1984). – ($N_c = 0$)
T. Appelquist et al., Critical Behavior in $(2+1)$-Dimensional QED, Phys. Rev. Lett. 60, 2575 (1988).
D. Nash, Higher-Order Corrections in $(2+1)$-Dimensional QED, Phys. Rev. Lett. 62, 3024 (1989).
A. Kotikov, Critical behavior of 3-D electrodynamics, JETP Lett. 58, 731 (1993).
V. Gusynin et al., $2+1)$- dimensional QED with dynamically massive fermions in vacuum polarization, Phys. Rev. D 53, 2227 (1996).
Most papers (except Pisarski) give in the leading $1/N$ approximation

$$m = e^2 \exp\left(-\frac{c}{\sqrt{\frac{N_c}{N}-1}} + b\right), \quad N < N_c.$$

Dyson-Schwinger equations in QED



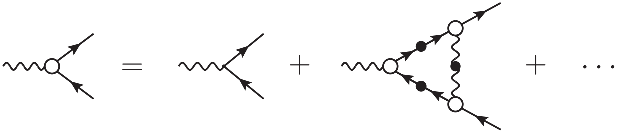
$$S^{-1}(p) = S_0^{-1}(p) + e^2 \int \frac{d^3q}{(2\pi)^3} \gamma_\mu S(q) \Gamma_\nu(q,p) D_{\mu\nu}(q-p),$$

$$D_{\mu\nu}^{-1}(p) = D_{0,\mu\nu}^{-1}(p) - Ne^2 \int \frac{d^3q}{(2\pi)^3} \mathrm{tr}\left[\gamma_\mu S(q) \Gamma_\nu(q,p-q) S(p-q)\right],$$

In the lowest order in $1/N$ the kernel is given by the diagram with exchange of one photon

$$K^{(2)}_{\beta'\alpha';\beta\alpha}(k,k,k-p) = (ie)^2 \Gamma^\mu_{\alpha\alpha'}(p,k;p-k) \Gamma^\nu_{\beta'\beta}(k,p;k-p) D_{\mu\nu}(p-k).$$

DSE for the vertex in the so-called three-gamma (Landau) approximation



We have a closed system of equations for S, G, Γ.

Dressed fermion and photon propagators:

$$S(p) = \frac{1}{-i\hat{p}A(p) + B(p)} = \frac{Z(p)}{-i\hat{p} + \Sigma(p)},$$
$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)\frac{1}{p^2[1+\Pi(p)]},$$

For the mass function $\Sigma(p) = B(p)/A(p)$ we get the integral equation

$$\Sigma(p) = \int\limits_0^\infty \frac{dk k^2 \Sigma(k)}{k^2 + \Sigma^2(k)} K(p,k;\Sigma^2).$$

$$K(p,k) = \frac{1}{4A(p)A(k)} \int \frac{d\Omega_k}{(2\pi)^3} (\gamma_5)_{\beta\alpha} K_{\beta'\alpha';\beta\alpha}(k,k,k-p;\Sigma^2)(\gamma_5)_{\alpha'\beta'},$$

Near the phase transition point the function $\Sigma(p)$ is small and the kernel simplifies

$$K(p,k) \simeq \left[\frac{4(2+\xi)}{\pi^2 N}\left(1-\frac{c}{\pi^2 N}\right)+\frac{8(20-8\xi+3\xi^2)}{9\pi^4 N^2}\frac{\min(p^2,k^2)}{\max(p^2,k^2)}\right] \times \frac{1}{\max(p,k)}\frac{A^2[\max(p,k)]}{A(p)A(k)},$$

$$A(p) \simeq \left(1+\frac{16}{9\pi^2 N}\right)\left(\frac{p}{\alpha}\right)^{-2\gamma},$$

where the anomalous dimension $\gamma = 2(3\xi-2)/(3\pi^2 N)$. Due to the scale invariance we look for a powerlike solution for the mass function $\Sigma(p) \sim p^{-b}$, and for the exponent b we obtain to the order $1/N^2$ the equation

$$b(1-b) = \frac{32}{3\pi^2 N} + \frac{64(3\pi^2-44)}{9\pi^4 N^2}.$$

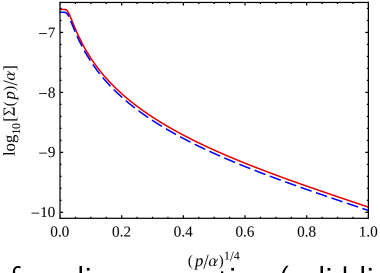
Chiral symmetry breaking occurs when b becomes complex, which determines the gauge invariant critical value $N_c = 2.85$ so that the symmetry is broken for $N < 3$.

Dynamical mass function (approximate analytical solution)

$$\Sigma(p) = \Sigma_0 F\left(\frac{1+i\nu}{4}, \frac{1-i\nu}{4}; \frac{3}{2}; -\frac{p^2}{\Sigma_0^2}\right), \quad \nu = \sqrt{4\lambda - 1},$$

$\lambda = \frac{32}{3\pi^2 N} + \frac{64(3\pi^2-44)}{9\pi^4 N^2}$. Dynamical mass

$$m \equiv \Sigma_0 = \alpha \exp\left(-\frac{2\pi}{\nu} + \delta + \frac{2\tan^{-1}\nu}{\nu}\right), \quad \alpha = e^2 N/8.$$



Numerical solution of nonlinear equation (solid line) versus an approximate analytical solution (dashed line) at $N = 2$.

Chiral condensate

The chiral condensate

$$\langle\bar{\psi}\psi\rangle = -\frac{2\alpha}{\pi^2\lambda}\Sigma(p=\alpha).$$

For $N = 2$ we get an estimate

$$\frac{\langle\bar{\psi}\psi\rangle}{e^4} \approx -4.64 \times 10^{-12}.$$

The condensate value is very small because $N = 2$ is close to the critical $N_c = 2.85$. V.P. Gusynin, P.K. Pyatkovskiy, Critical number of fermions in three-dimensional QED, Phys. Rev. D 94, 125009 (2016).
At present lattice simulations give only an upper bound for the chiral condensate $\langle\bar{\psi}\psi\rangle/e^4 < -8\cdot 10^{-8}$ - C. Strouthos and J. Kogut, Proc. Sci, LAT2007 (2007).

Infrared divergences in conventional perturbation theory in e.

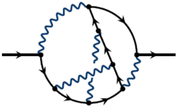
The first divergence appears in the fermion self-energy at two loops due to fermion loop ($\sim \ln(e^2/p)$) – Jackiw, Templeton (1981), Guendelman (1983).

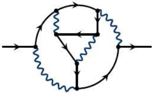
Is the fermion self-energy finite or not in quenched (without fermion loops) approximation?

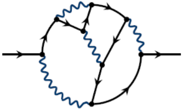
$$S(p,\xi) = \frac{i}{\hat{p}(1-\sigma(p,\xi))}, \; \sigma(p,\xi) = \sum_{m=1}^{\infty} \sigma_m(\xi,\epsilon) \left(\frac{\alpha}{2\sqrt{\pi}\,p}\right)^m \left(\frac{\overline{\mu}^2}{p^2}\right)^{m\epsilon}.$$

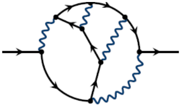
Dimensional regularization $d = 3 - 2\epsilon$.

In quenched QED at one, two, three, and four loops we encountered 1, 2, 10, and 74 fermion self-energy diagrams.

$2 *$  $= \frac{\pi^2(10-\pi^2)}{\varepsilon}$

$2 *$  $= -\frac{\pi^2(480-49\pi^2)}{18\varepsilon}$

$2 *$  $= -\frac{\pi^2(480-49\pi^2)}{18\varepsilon}$

$2 *$  $= -\frac{\pi^2(480-49\pi^2)}{18\varepsilon}$

Divergent four-loop diagrams in Landau gauge ($\xi = 0$).
Three-loop correction is finite and gauge invariant but the four-loop one has singularities except in the Feynman gauge where it is finite.
Pikelner, V.G., Kotikov, Teber, PRD 102, 105012 (2020).
Up to four loops, gauge-dependent terms are completely determined by lower order ones in agreement with the Landau-Khalatnikov -Fradkin transformation. V.G., Kotikov, Teber, PRD 102, 025013 (2020).

Дякую за увагу!