Electron cooling theory of antiprotons

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Memorandum of Understanding

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Memorandum of Understanding

between

and

– both

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Preamble

The Nuclear Physics Institute at JÜLICH (IKP-4) and the Institute for Applied Physics at the National Academy of Sciences of Ukraine discussed on several occasions the basic possibilities of cooperation for research on electron cooling of antiprotons for the High-Energy Storing Ring (HESR) as a part of the Facility for Antiproton and Ion Research (FAIR) project (hereinafter called the "PROJECT"). JÜLICH and IAP have developed in these discussions a

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9. The exclusive place of jurisdiction for all disputes arising from this Agreement shall be Jülich.

Jülich, 18 July 2012

Sumy, 14.08.2012

Forschungszentrum Jülich GmbH

Institute of Applied Physics of the National Academy of Sciences of Ukraine

i.V. R. Maier   i.V.

i.V. Prof. R. Maier
Head of IKP-4

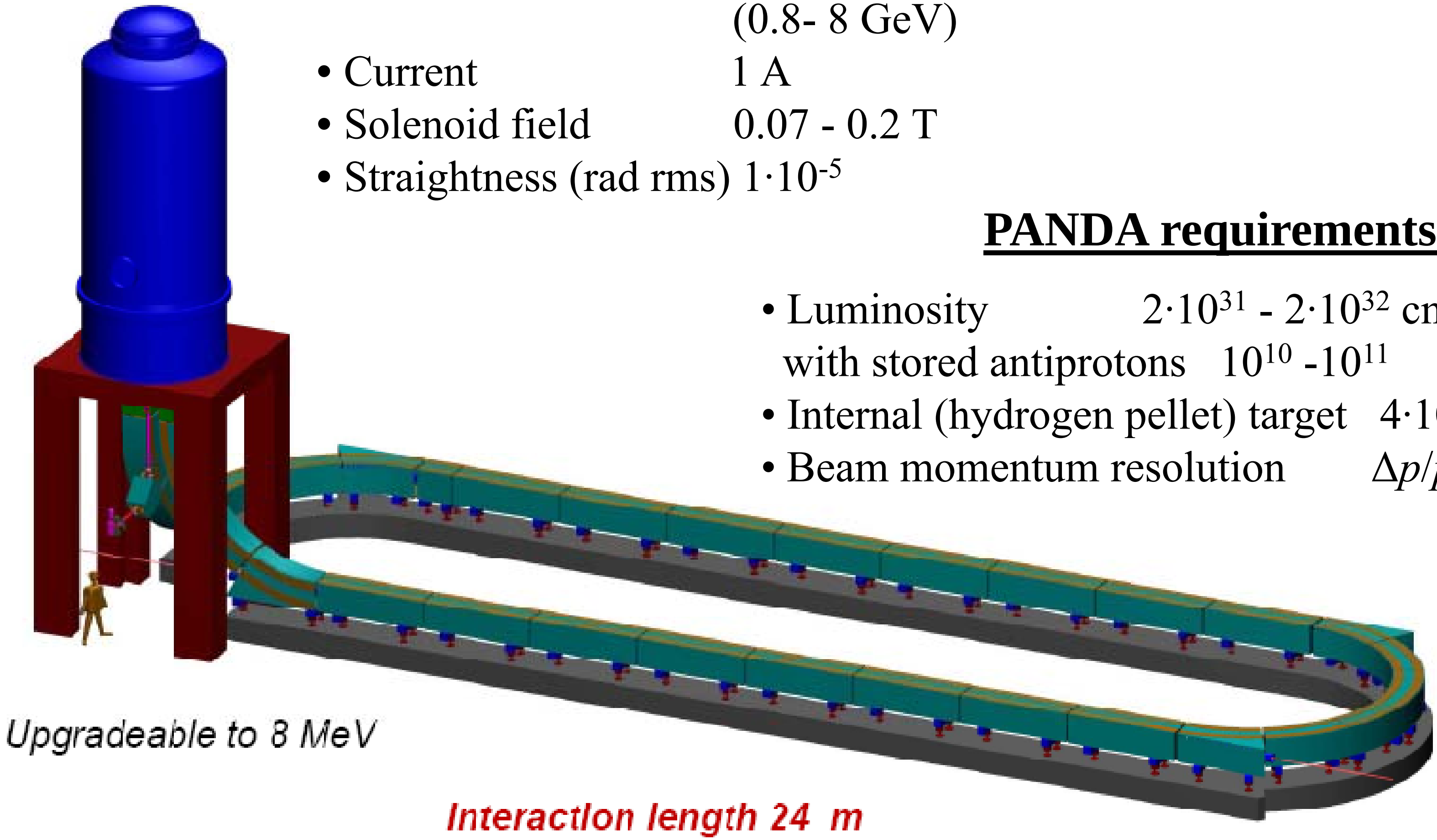
i.V. K. Jürgensen
Legal department JÜLICH

Prof. Volodymyr Yu. Storizhko
Director of the Institute of Applied Physics of NASU

Electron cooling

HESR Electron Cooler

- Electron energy 0.45-4.5 MeV
  (0.8- 8 GeV)
- Current 1 A
- Solenoid field 0.07 - 0.2 T
- Straightness (rad rms) $1 \cdot 10^{-5}$



PANDA requirements

- Luminosity $2 \cdot 10^{31}$ - $2 \cdot 10^{32}$ cm^{-2} s^{-1}
  with stored antiprotons 10^{10} - 10^{11}
- Internal (hydrogen pellet) target $4 \cdot 10^{15}$ cm^{-2}
- Beam momentum resolution $\Delta p/p = 10^{-5}$.

B. Galnander, HESR-meeting, GSI, 2008-04-08

Friction force measurement in electron cooling
(Budker Institute of Nuclear Physics)

N. S. Dikanskii et al., Zh. Eksp. Teor. Fiz. 94, 65-73, (1988)

Fig.1 Diagram of apparatus (MOSOL)

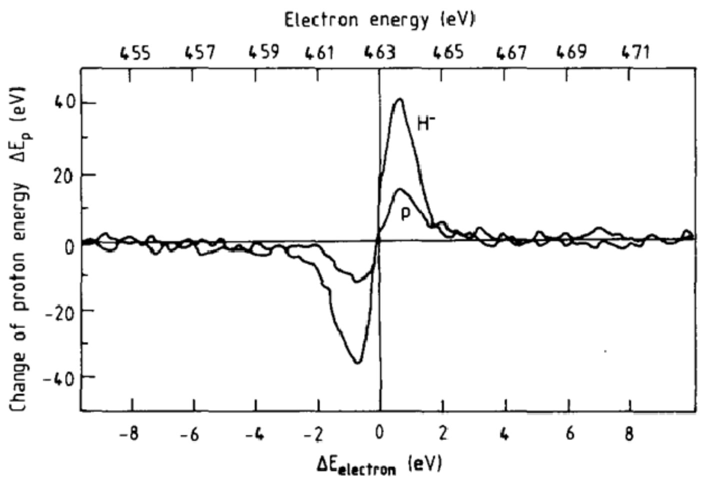


Fig.2 Change of energy of ions of different signs as a function of electron energy

Energy of the hydrogen ions, keV	850
Stability of the ion energy, keV	$\pm 2,5\cdot 10^{-5}$
Electron energy, eV	463
Electron current, mA	$1-15$
Electron density, $10^{8}\,cm^{-3}$	$1,6-23,5$
Magnetic field, kG	$1-3$
Parallelism of the magnetic field, $B_{\perp}/B_{0}$	$5\cdot 10^{-5}$
Length of the cooling section, m	2,4

Qualitative explanation of the difference in the friction forces

The known analytical expressions of energy losses of a charged particle moving in an electron gas are identical for positive and negative charged particles.

$$-\frac{d\varepsilon}{dt} \sim q^2 \qquad (1)$$

Longitudinal friction force
of the negative charged particle

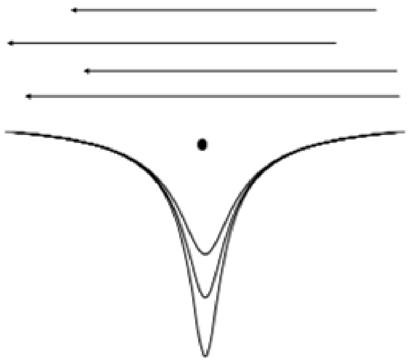
$$F_{\parallel} = -\frac{2\pi n e^4}{m\upsilon^2}\frac{2\upsilon_{\perp}^2\upsilon_{\parallel}}{\upsilon^3}L_C + \Delta F_{\parallel} \qquad (2)$$

Additional contribution in the friction force of the negative charged particle

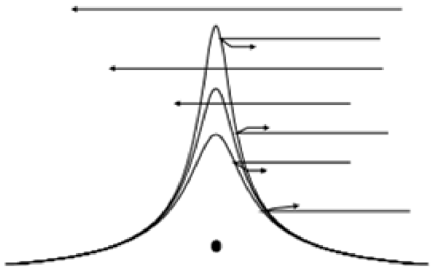
$\rho < \rho_{\min}$

$$\Delta F_{\parallel} = -\pi\rho_{\min}^2 n\upsilon \cdot 2m\upsilon, \qquad (3)$$

$$L_C = \ln\frac{\rho_{\max}}{\rho_{\min}}, \quad \rho_{\min} = \frac{2e^2}{m\upsilon^2}$$

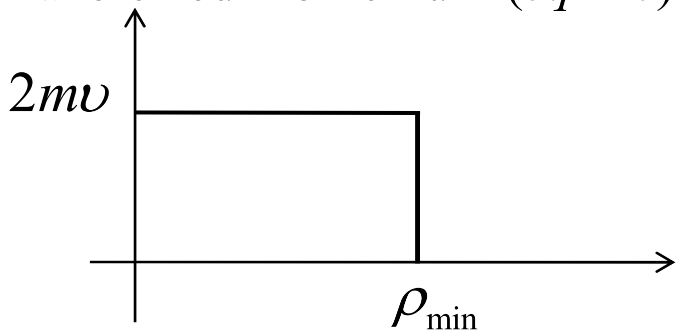


$Z > 0$: no scattering

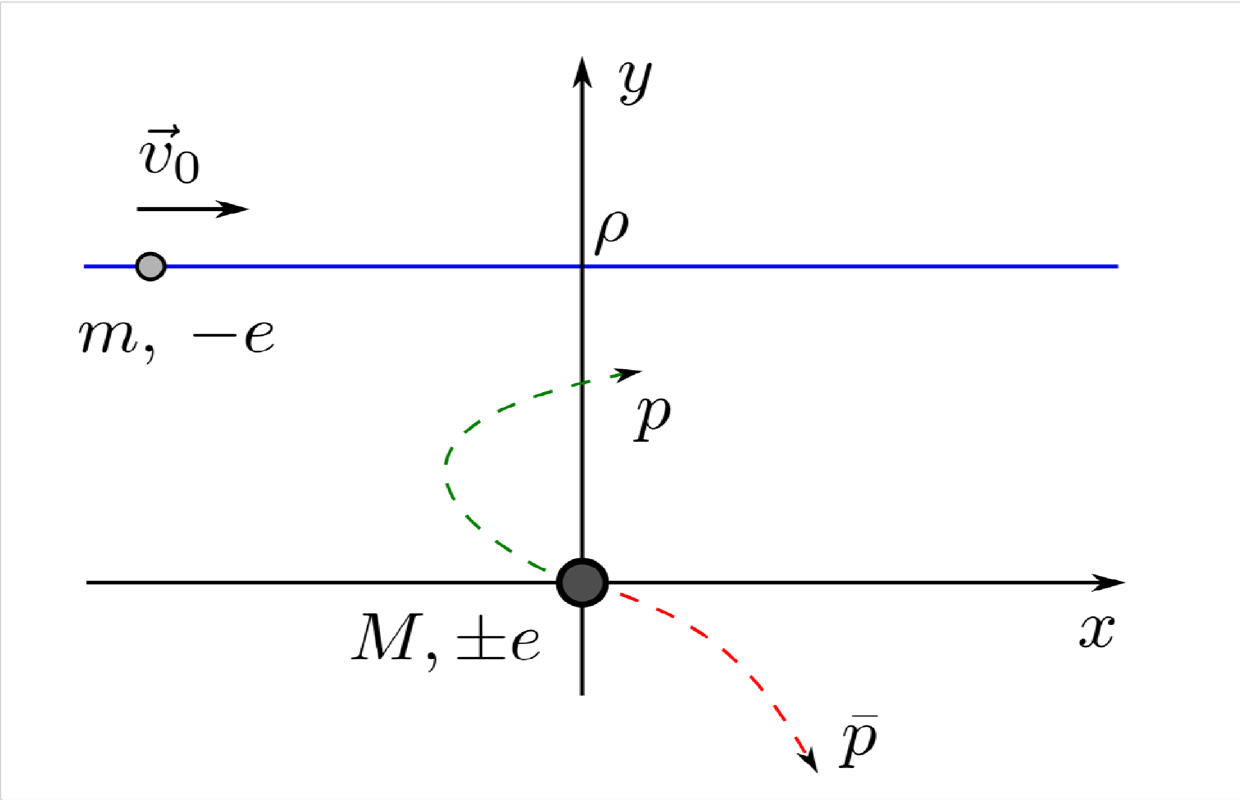


$Z < 0$: backscattering

Transferred momentum $(eq > 0)$



One-dimensional motion of e^-



The motion of an electron in an extremely strong magnetic field:

- Electron on a string
- String does not affect the longitudinal movement of e^-

Transferred energy

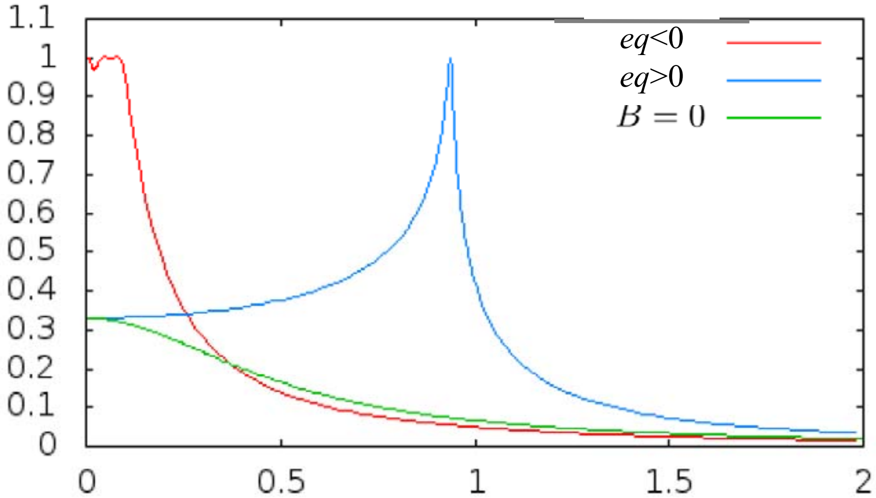
Accompanying reference system

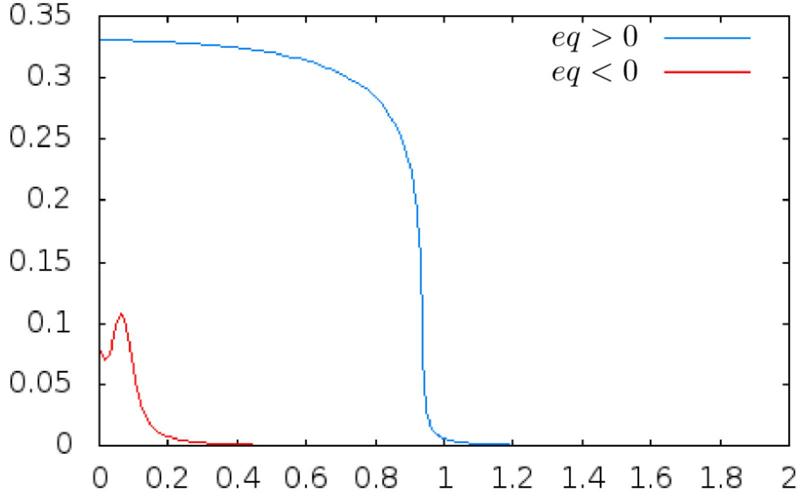
(at $t = 0$: $\upsilon_e = \upsilon_0$, $\upsilon_p = 0$)

$\mu = 0.1$

Laboratory reference system

(at $t = 0$: $\upsilon_e = 0$, $\upsilon_p = \upsilon_0$)

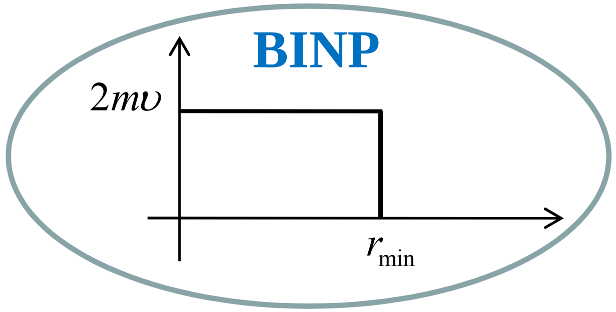




ρ / ρ_{min}

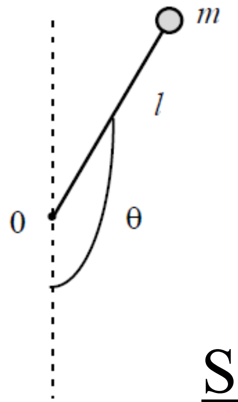
Galilean transformation

$$\begin{cases} x = x' + Vt' \\ \upsilon = \upsilon' + V \\ p = p' + mV \\ E = E' + p'mV + mV^2/2 \end{cases} \qquad (4)$$

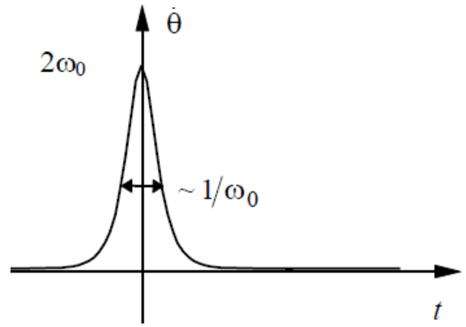


Nonlinear mathematical pendulum

Equation of motion



$$\dot{\theta} = \pm 2\omega_0 \cos\frac{\theta}{2} \qquad (5)$$



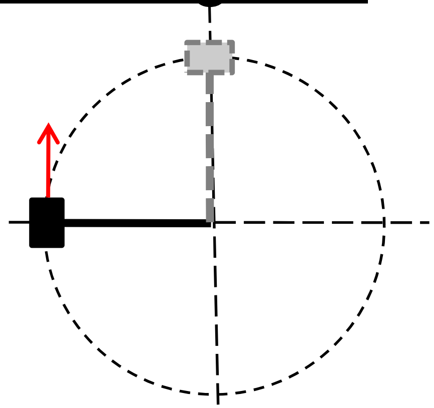
Soliton-like solution

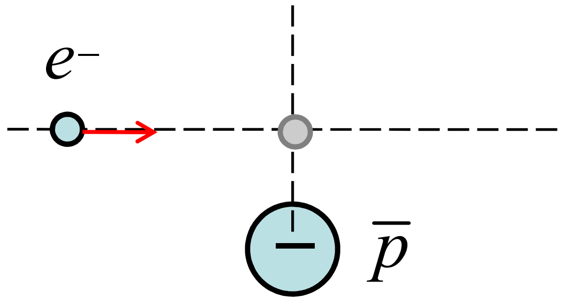
$$\dot{\theta} = \frac{\pm 2\omega_0}{\mathrm{ch}(\omega_0 t)} \qquad (6)$$

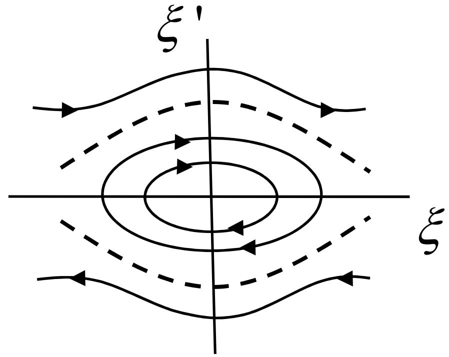
A nonlinear mathematical pendulum has soliton-like solution on the boundary of the vibration and rotation modes.

Mathematical pendulum and movement in strong magnetic field

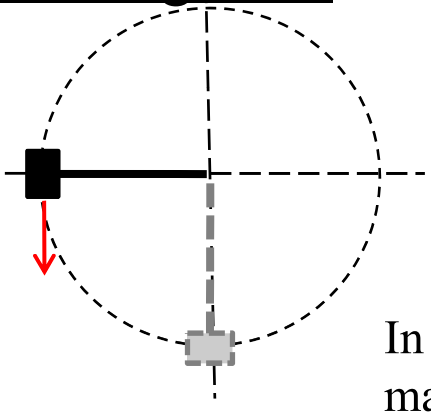
Scattering at H^-

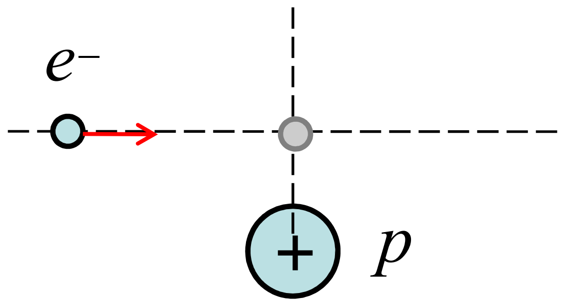


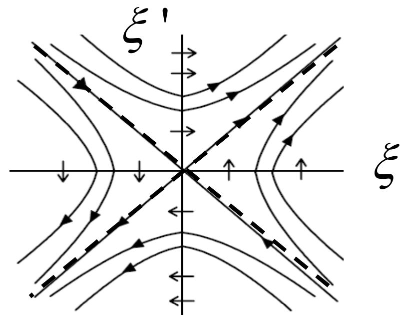




Scattering at H^+







In the vicinity of separatrix, the electron motion in strong magnetic field near the charged particle has a soliton-like character.

Quantum Field Theory Approach

Scattering is a transition $a,n \longrightarrow a',n'$ particle-plasma system

Scattering matrix $$S = T\exp\{-i\int V(t)dt\} \approx 1 - i\int V(t)dt$$

1[st] Born approximation

Hamiltonian $$H = H_0 + V\ ,\ V = q\phi(\vec{r},t)$$

Probability $$W_{if} = 2\pi \cdot \delta(E_f - E_i) \cdot |< a',n' | V | a,n >|^2$$

Averaging and summation over the states of medium $n,\ n'$

$$W_{a,a'} = \sum_n e^{\beta(\Omega + \mu N_n - E_n)} \sum_{n'} W_{if}$$

$$\beta = \frac{1}{T}$$

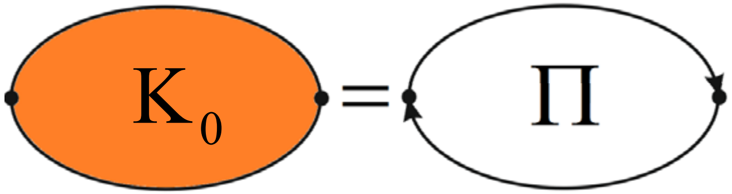
Energy loss
$$-\frac{dE}{dt} = \sum_{a'} (\varepsilon_a - \varepsilon_{a'}) W_{aa'} \qquad (7)$$

Energy loss of a charged particle in the first Born approximation

Total probability of transition

$$W_{\vec{k}} = \frac{2V_{\vec{k}}^{2}}{1-e^{-\beta\omega}} \operatorname{Im} \frac{\Pi\left(\vec{k},\omega\right)}{1-V_{\vec{k}}\Pi\left(\vec{k},\omega\right)} \qquad (8)$$

Feynman diagram of the Green's function in one-loop approximation



Energy loss of a charged particle in an electron gas

$$-\frac{d\varepsilon}{dt} = \frac{4\pi n q^{2} e^{2}}{m\upsilon} \ln\left(\frac{2Mm\upsilon^{2}}{(M+m)\hbar\omega_{p}} \right) \qquad (9)$$

I.A.Akhiezer// ZhETF. 40, 954, (1961), Sov.Phys.JETP 13, 667,(1961)

The second Born approximation

Probability of the transition in the second Born approximation

$$W_{\vec{k}} = 2\pi \left\{ V_{\vec{k}}^2 \Phi_1\left(\vec{k},\omega\right) + \sum_{\vec{k}_1} V_{\vec{k}} V_{\vec{k}-\vec{k}_1} V_{\vec{k}_1} \Phi_2\left(\vec{k},\vec{k}_1,\omega,\omega_1\right) \right\} \quad (10)$$

where $V_{\vec{k}}$ is the Fourier component of the interaction potential

$$\Phi_1\left(\vec{k},\omega\right) = \sum_{mn} \omega_n M_{nn}^{(1)} \delta\left(E_m - E_n - \omega\right)$$

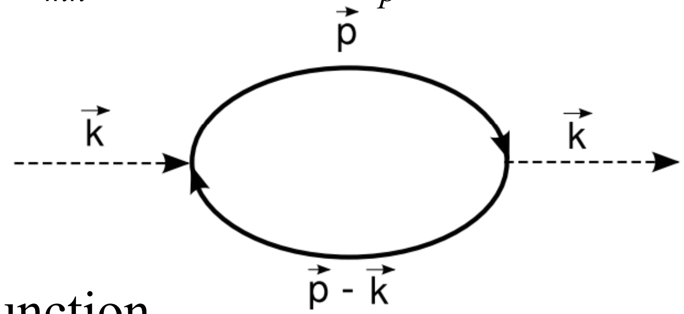
$$\Phi_2\left(\vec{k},\vec{k}_1,\omega,\omega_1\right) = \sum_{n,m,l} \omega_n \frac{M_{nn}^{(2)}}{E_n - E_l + \omega_1} \delta\left(E_m - E_n - \omega\right)$$

where $\omega_n = e^{\beta\left(\Omega + \mu N_n - E_n\right)}$, $M_{nn}^{(1,2)}$ is the matrix elements in the first and second Born approximation.

The second Born approximation

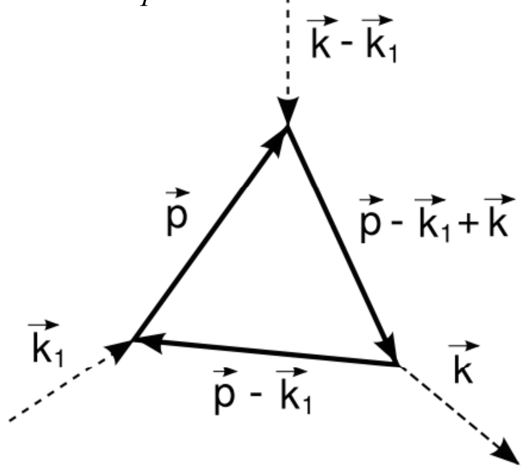
Two-particle Green's function

$$K\left(\vec{k},t\right) = -i\sum_{mn}\omega_n e^{i\left(E_n - E_m\right)t}\sum_{\vec{p}}\left\langle n\right| a^{+}_{\vec{p}-\vec{k}} a_{\vec{p}} \left|m\right\rangle \left\langle m\right| a^{+}_{\vec{p}} a_{\vec{p}-\vec{k}} \left|n\right\rangle \quad (11)$$



Three-particle Green's function

$$G\left(\vec{k}_1,\vec{k},t_1',t_2'\right) = \sum_{n,l,m}\omega_n e^{i\left(E_n - E_l\right)t_1'} e^{i\left(E_n - E_m\right)t_2'}\sum_{\vec{p}}\left\langle n\right| a^{+}_{\vec{p}-\vec{k}_1} a_{\vec{p}+\vec{k}-\vec{k}_1} \left|m\right\rangle \left\langle m\right| a^{+}_{\vec{p}+\vec{k}-\vec{k}_1} a_{\vec{p}} \left|l\right\rangle \left\langle l\right| a^{+}_{\vec{p}} a_{\vec{p}-\vec{k}_1} \left|n\right\rangle \quad (12)$$



Why QFT for EC?

Two types of quantum effects in magnetic field

1) Electron beam is a quantum object, when $T < T_0$

for EC

$$N \approx 3 \cdot 10^{7}\, cm^{-3} \qquad \omega_B \approx 3.5 \cdot 10^{10}\, c^{-1}$$

$$\omega_p \approx 2.9 \cdot 10^{8}\, c^{-1} \qquad \hbar\omega_B \approx 2 \cdot 10^{-5}\, eV$$

degeneration temperature

$$T_0 = \frac{\hbar^2}{2m}(3\pi^2 N)^{2/3} \sim 10^{-10}\, eV$$

2) Electron is on Landau level, when $T < \hbar\omega_B$

$$\frac{\hbar\omega_B}{T_\perp} = \frac{eB\,\hbar}{2mcT_\perp} \sim 10^{-5},$$

$$T_\perp \sim 1\, eV$$

$$B \approx 2 \cdot 10^{3}\, Gs$$

Main advantage of QFT

- absence of any phenomenological constants
(all the classical approaches give a logarithmic divergence)
- development beyond perturbation theory

THANK YOU FOR ATTENTION!